COMPUTER-AIDED DETERMINATION OF RESONATOR CHARACTERISTICS BASED ON EXPANSION IN NORMAL MODES AND USING AUTOMATIC NETWORK ANALYSER DATA

Q-6

Donald W. Griffin
Department of Electrical Engineering
University of Adelaide
South Australia 5001

ABSTRACT

Errors that are likely to arise in determining resonator characteristics by simple equivalent circuit interpretation of measured input impedance data are explained. A method based on expansion in normal modes and involving computer aided analysis of data gathered with an automatic network analyser is explained by reference to experimental evaluation of microstrip-type rectangular patch antenna element characteristics.

Introduction

The speed and accuracy with which computer-controlled automatic network analysers can measure microwave resonator response, correct the data for measuring instrument imperfections and then process it to derive such parameters as the resonant frequency f_0, the coupling factor β and the quality factors Q_0, Q_{ext} and Q_L, allows the engineer to focus effort onto the design of the resonator and the assembly of which it forms part. Miniaturization of microwave components in general, together with widespread application of microstrip in practical assemblies, has resulted in a large proportion of the resonators, that are incorporated in designs, having Q-factors that are less than one hundred. It is not commonly realised that the practical interpretation of microwave resonator input impedance measurements [1] in terms of the behaviour of equivalent circuits with fixed elements at detuned-short or detuned-open positions can lead to significant numerical errors, and even misinterpretation, if the Q is in the low to moderate range, $10 < Q < 100$.

RLC circuit impedance plot on a Smith chart

The impedance of an equivalent circuit, comprising a parallel combination of resistance R, inductance L and capacitance C, as a function of excitation frequency follows a perfectly circular locus on the Smith chart. The locus approaches a point of tangency within the perimeter of the chart at frequencies well below and well above resonance and the position of this point corresponds to zero (normalized) impedance on the chart or the so-called detuned short position A on Figure 1. The coupling factor β is obtained as the intercept of this impedance locus with the real axis scaled in normalised resistance values and the various quality factors are obtained from intercepts with $r=\pm x$, $b=\pm 1$ and $b=\pm(g+1)$ contours as shown in Figure 1.

The impedance of a series combination of R, L and C also follows a perfectly circular locus which approaches a point of tangency diametrically opposite A corresponding to infinitely large (normalized) impedance and called the detuned open position. That point of tangency is a quarter of a wavelength from the point A around the chart.

Microwave resonator impedance behaviour

Microwave resonators usually incorporate lengths of uniform cross-section guiding structure terminated in approximations to short- or open-circuit conditions. The input impedance of a short-circuit terminated line of length ℓ is given by,

$$Z_{in}/Z_0 = \tanh \gamma\ell \qquad (1)$$

and the input admittance of an open-circuit terminated line of the same length is given by,

$$Y_{in}/Y_0 = \tanh \gamma\ell \qquad (2)$$

where Z_0 is the wave impedance of the line, $Y_0=1/Z_0$ is the wave admittance and γ is the propagation constant. The impedance behaviour of resonators formed from stubs of this type exhibits features pertaining to this type of hyperbolic tangent function and includes more than one resonance. If the cross-sectional dimensions are not small compared with a wavelength then modes other than TEM may exist and give rise to further resonances. These modes have quite different Z_0 and γ values but still have stub impedance characteristics given by (1) or (2).

The impedance locus of a typical microwave resonator on a Smith chart is not a perfect circle over a frequency range that includes a resonance and deviation from the lumped circuit behaviour shown in Figure 1 is greater for frequencies that are further from resonance. The input impedance can be represented by a summation over all resonant modes [6] as,

$$Z_{in}/Z_0 = \sum_a \left[1/Q_{ext,a}\right] \Big/ \left[j(\omega/\omega_a) - j(\omega_a/\omega) + (1/Q_a)\right] \qquad (3)$$

and may be approximated at the a'th resonance by the expression,

$$Z_{in}/Z_0 = Z_a/Z_0 + \left[1/Q_{ext,a}\right] \Big/ \left[j(\omega/\omega_a) - j(\omega_a/\omega) + (1/Q_a)\right] \quad (4)$$

The first term on the right hand of (4) is the contribution of all of the other resonances of the cavity in the vicinity of the resonance that is approximated by (4). It is clear that impedance behaviour does not start and finish at a point such as A on the perimeter of the Smith chart.

The input impedance locus measured on a microstrip rectangular patch antenna element is shown in Figure 2 by the broken line curve. Extending the frequency range of measurement would reveal other resonances in other regions of the Smith chart. The residual effects of those resonances contribute to the gap between the measured locus and the perimeter of the Smith chart.

Errors due to equivalent circuit interpretation of results

The errors that may arise, in applying the classical resonant equivalent circuit approach of interpreting the experimental results for practical resonators, can be illustrated by considering the input impedance of a microstrip-type rectangular patch antenna element shown in Figure 2. A computer-aided design approach has been developed, according to the method detailed in Ginzton [1] and other texts [2], [3], [4], [5], by determining the circle of best fit for the impedance data stored in the computer-controller of the network

 0149-645X/82/0000-0331 $00.75

analyser, shown as CBF in Figure 2, rotating that circle about the centre of the Smith chart to the "correct" detuned short position to give RCBF in Figure 2 and then interpreting the gap between the circle of best fit and the perimeter of the Smith chart as coupling structure loss, in accordance with the treatment in Ginzton [1], pp. 424-428.

Errors arise because the circle of best fit is not the correct circle to use, it should not be rotated through a length of line to the "correct" detuned short position but instead should have equivalent inductive reactance X_L subtracted from it to place the locus of results symmetrically about the real axis, and most importantly, the gap is a consequence of the fundamental difference between distributed parameter resonators compared with a fixed element equivalent circuit and does not necessarily indicate coupling structure loss. Difficulty is usually experienced in determining the correct detuned short position. From Figure 2 it would appear to correspond to rotation from the position on the Smith chart perimeter adjacent to the point where the experimental results cross over. Determining the circle of best fit also presents difficulties. Erroneous data points should be rejected without making the computational procedure for determining CBF too lengthy.

Method based on expansion in normal modes

A more fundamental approach to the interpretation of the measured results based upon expansion in normal modes is outlined in Slater [6] and can be used as the basis for computer-aided design software warranted by the accuracy and the frequency range over which data can be readily measured. Figure 3 illustrates the way in which the input impedance results for the microstrip rectangular patch antenna analysed in Figure 2 should be graphically treated if this more fundamental approach is followed.

First, the locus of measured results is translated by subtracting an inductive reactance X_L, justified as the reactive part of Z_a/Z_0 of expression (4) and selected in magnitude so that a position of symmetry about the real axis of the Smith chart is achieved as shown in Figure 3. Second, a decision is required on the procedure that will be adopted in processing the translated set of results. A simple procedure that yields reasonable accuracy is to assume that a circle through the resonance and z=0 fits the results in the vicinity of resonance. Alternatively the actual results can be interpolated and intersections with $r=\pm x$, $b=\pm 1$ and $b=\pm(g+1)$ contours determined. A more elaborate procedure would be to fit the appropriate theoretical expression for cavity impedance involving tanh $\gamma\ell$ factors to the experimental results and proceed to obtain Q values from the intersections between this fit and the $r=\pm x$, etc., contours.

Whichever procedure is adopted the coupling factor is obtained from the intersection of the results with the real axis of the Smith chart. Resonance and other frequencies are best interpolated on the linear frequency scale obtained on any line parallel to the imaginary axis of the chart. If the final result is required in the form of an equivalent RLC circuit then the circle of fit shown on Figure 3 is the appropriate procedure and the third step is to determine the equivalent circuit elements for that circle. If the experimental results span more than one resonance then, provided they do not overlap extensively in the way that slightly detuned degenerate modes would, each loop can be treated separately according to the procedure set out above.

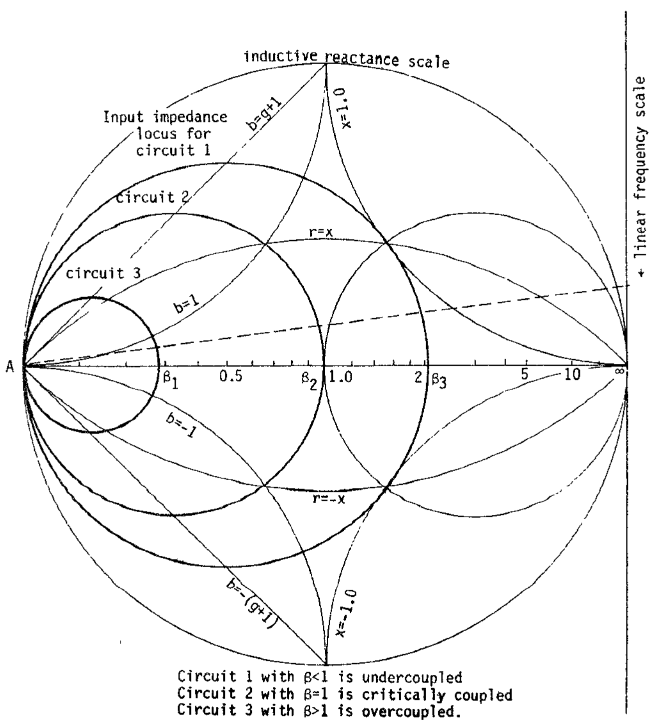

Figure 1. The input impedance response of three different RLC equivalent circuits plotted on a Smith chart. Intercepts of the circular loci with $r=\pm x$ contours yield Q_0, with $b=\pm 1$ contours yield Q_{ext} and with $b=\pm(g+1)$ lines yield Q_L.

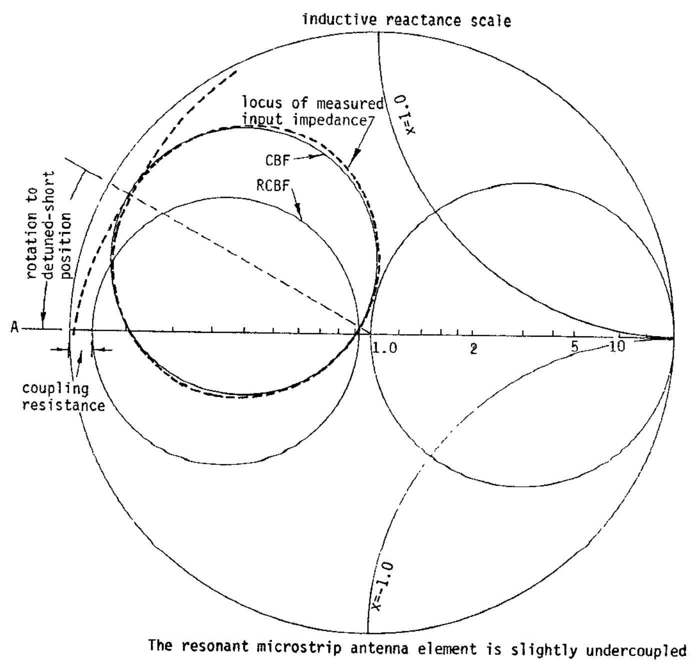

Figure 2. Interpretation of resonator type input impedance results for a microstrip rectangular patch antenna element using the resonant equivalent circuit.

Comparison of the common and the normal mode method

As an example of how errors that are quite large may arise Figure 4 illustrates the commonly used practical method and the method advocated in this paper. The experimental results are shown translated to a position of symmetry about the real axis. The rotated circle of best fit (RCBF) obtained according to the commonly used practical method does not pass through A and the gap is interpreted as arising from coupling structure resistance of 2.5 ohms. Hence RCBF is shifted to pass through A and Q values are evaluated from intersections along this shifted circle. Compared with the normal mode expansion method illustrated on the same Figure errors in Q's of about 12 percent result and are incurred if the common practical method is employed. Specifically, Q_0 is evaluated according to the common method by reading frequencies f_1 and f_1' corresponding to the intersections of the shifted RCBF circle with the r=±x contours and calculating $(f_1'-f_1)/f_0$. The normal mode method uses the intersections of the experimental results, or the circle of fit through z=0 and resonance, with the r=±x contours. The frequencies corresponding to these intersections are f_2 and f_2' and $Q_0=(f_2'-f_2)/f_0$ is about 12% larger.

Conclusion

The accuracy and close frequency spacing of measurements of the input characteristics of microwave resonators, that are now possible using computer controlled network analysers, warrants more accurate analysis of results than that achieved by the commonly used method. A procedure that takes account of the residual effect of other resonances is advocated and the types of error that can be avoided by its use are illustrated. The instrumentation controller can be used as the computer for automating the analysis which involves curve fitting techniques and determination of intersection points. Software for automating the commonly used and the normal mode methods has been developed for use on the HP9845 automatic network analyser controller.

References

1. GINZTON, Edward L., "Microwave measurements", (McGraw-Hill, New York, 1957), Chapter 9, pp.391-428.
2. ALTMAN, Jerome L., "Microwave circuits", (D. Van Nostrand, Princeton, 1964), Chapter 5, pp.213-231.
3. MONTGOMERY, C.G., DICKE, R.H. AND PURCELL, E.H., "Principles of microwave circuits", (McGraw-Hill, New York, 1948), Chapter 7, pp.214-234.
4. BARLOW, H.W. and CULLEN, A.L., "Microwave measurements", (Constable, London, 1950), pp.256-274.
5. GRIVET, P. (translated by HAWKES, P.W.), "Microwave circuits and amplifiers",(Academic Press, London, 1976) Chapter 4, pp.135-191.
6. SLATER, John C., "Microwave electronics", (D. Van Nostrand, Princeton, 1950), Chapter 5, pp.89-95.

Acknowledgements

Support from the Radio Research Board, Australia for this work is gratefully acknowledged along with stimulating discussions with N.M. Martin, P. Shields and A.J. Yates. Support for related work from the RADC Post-Doctoral Program via Syracuse University and SCEEE, St. Cloud, FL. is also acknowledged.

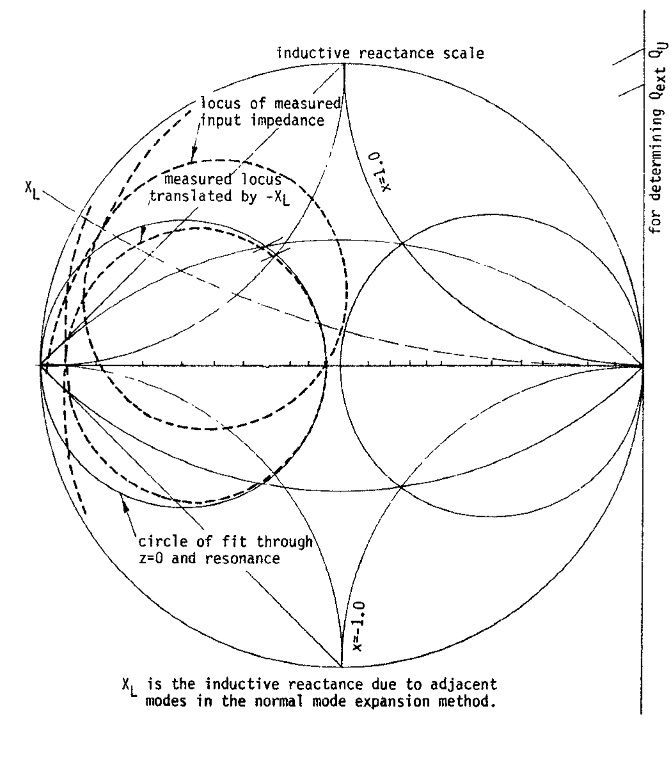

Figure 3. Interpretation of resonator type input impedance results for a microstrip rectangular patch antenna element using a method based on expansion in normal modes.

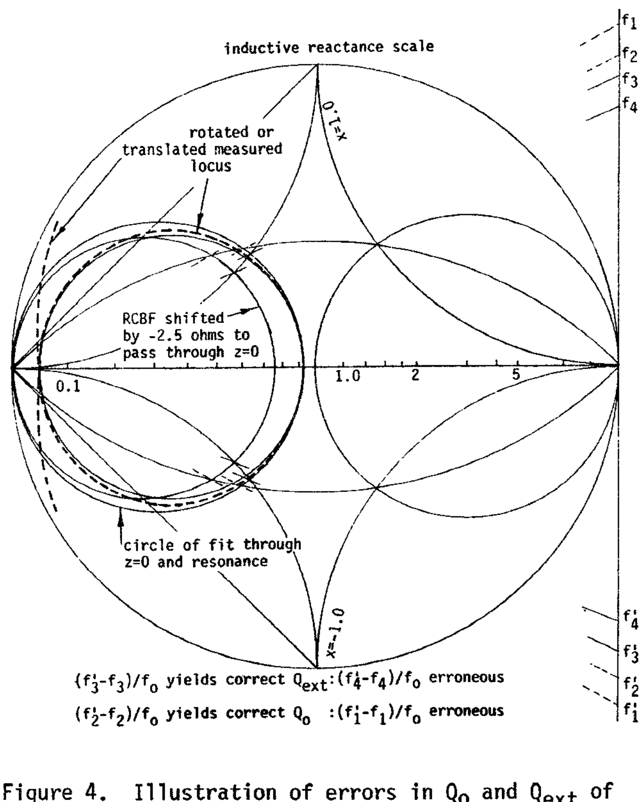

Figure 4. Illustration of errors in Q_0 and Q_{ext} of about 12% and erroneous coupling structure loss resistance of 2.5 ohms if the commonly used practical method is used instead of the method based on normal mode expansion.